The Simplest Possible Behavioral Finance Bubble Model

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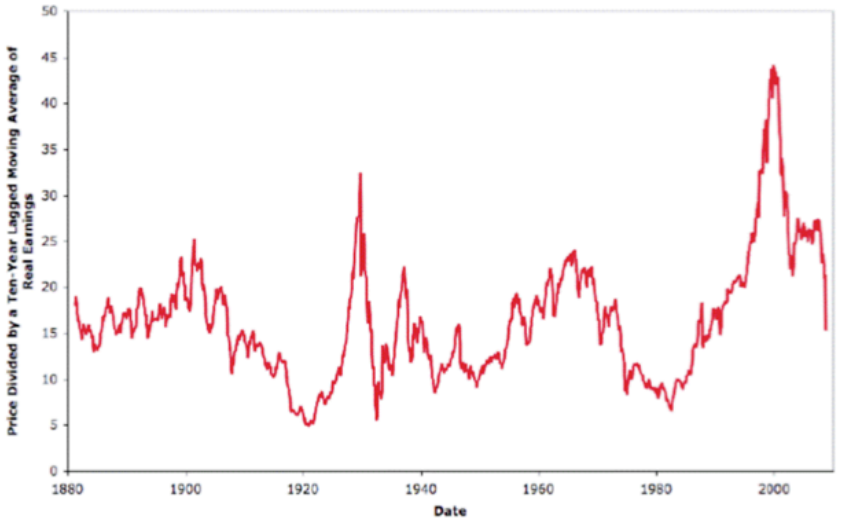
Begin with Charles Kindleberger paraphrasing Hyman Minsky:

> Hyman Minsky['s]… emphasis on the fragility of the monetary system and its propensity to disaster… a lineal descendant of a model, set out… by a host of classical economists including John Stuart Mill, Alfred Marshall, Knut Wicksell, and Irving Fisher…. [E]vents leading up to a crisis start with a "displacement," some exogenous, outside shock…. Displacement brings opportunities for profit… investment and production pick up. A boom is under way… fed by an expansion of bank credit… the formation of new banks, the development of new credit instruments, and the expansion of personal credit outside of banks…. [T]he urge to speculate is… transmuted into effective demand for goods or financial assets…. Prices increase, giving rise to new profit opportunities and attracting still further firms and investors. Positive feedback… "euphoria." Speculation for price

> increases is added to investment for production and sale… "overtrading."…
>
> Overestimation of profits comes from euphoria… requires no explanation. Excessive gearing arises from cash requirements that are low… buying on margin, or by installments, under circumstances in which one can sell the asset and transfer with it the obligation to make future payments…. "Monkey see, monkey do." In my talks about financial crisis… I have polished one line that always gets a nervous laugh: "There is nothing so disturbing to one's well-being and judgment as to see a friend get rich."… [S]peculation for profit leads away from normal, rational behavior to what has been described as "manias" or "bubbles."… A larger and larger group of people seeks to become rich without a real understanding of the processes involved….
>
> The specific signal that precipitates the crisis may be the failure of a bank or firm stretched too tight, the revelation of a swindle or defalcation by someone who sought to escape distress by dishonest means, or a fall in the price of the primary object of speculation…. In any case, the rush is on. Prices decline. Bankruptcies increase. Liquidation sometimes is orderly but may degenerate into panic as the realization spreads that there is only so much money, not enough to enable everyone to sell out at the top. The word for this state—again, not from Minsky—is revulsion. Revulsion against commodities or securities leads banks to cease lending on the collateral of such assets. In the early nineteenth century this condition was known as discredit. Overtrading, revulsion, discredit—all these terms have a musty, old-fashion flavor. They are imprecise, but they do convey a graphic picture...

This is, I think, broadly right. Certainly if we look at stock prices over the past century and a half we see manias followed by panics and crashes—most prominently 1929 and 2000.

Figure 1
Stock Prices Divided by a Ten-Year Moving Average of Earnings, 1880-2009



But how do we build a model of this? That is the task of this note: to build the simplest possible model of a stock market with bubbles—with manias, panics, and crashes.

Make ten assumptions:

1. There are no rational agents in the stock market—no agents, patient or impatient, who understand the price dynamics and attempt to profit from them.

2. There is a single unit of agents—index them by the interval [0,1].
3. Of these agents, in any period t a fraction and a number p_t of them buy stocks; the rest buy bonds.
4. Each stock-buying agent invests a single unit of wealth in stocks.
5. Stocks are inelastically supplied in an amount of one unit: thus p_t is the number and share of agents holding stocks and the amount of wealth invested in stocks and the price of stocks—all three.
6. Stocks pay a stochastic dividend: with a serially uncorrelated probability π the dividend d_t paid in period t is δ; with probability $1-\pi$ the dividend d_t paid in period t is 0.
7. Bonds pay a fixed rate of return r.
8. In each period an agent does what he or she did last period, except that…
9. …agents randomly encounter each other and compare rates of return; thus a number $p_t(1-p_t)$ of comparisons between stock and bond returns are made each period.
10. …a number equal to a parameter λ times the difference in rates of return switches from the lower to the higher-performing strategy for the next period for each agent who makes the comparison.

We would want to relax these assumptions: we would want to investigate the consequences of adding rational agents, of adding patient agents, of allowing the number of agents in the marketplace to vary, of allowing the amount they invest to vary, of allowing a more sophisticated dividend process, of allowing more than one-period lookback comparisons of returns, and—especially—of allowing an asymmetric portfolio strategy-conversion rule. But this

gives us a start: the simplest possible behavioral model of manias, panics, and crashes.

Together assumptions 1-10 provide us with a difference equation for p_t, which is the price of stocks/wealth invested in stocks/number and share of agents investing in stocks. That difference equation is:

(1) $$p_{t+1} = p_t + \lambda p_t (1 - p_t)\left[\left(\frac{p_t - p_{t-1} + d_t}{p_{t-1}}\right) - r\right], \text{ with : } d_t = \begin{cases} \delta \text{ with probability } \pi \\ 0 \text{ with probability } 1-\pi \end{cases}$$

Taking unconditional expectations:

(2) $$\mathrm{E}(\Delta p_{t+1}) = \lambda p_t (1 - p_t)\left(\frac{E(\Delta p_t) + \pi\delta}{p_{t-1}} - r\right)$$

So:

(3) $$\mathrm{E}(\Delta p_{t+1}) = 0 \text{ if } \mathrm{E}(\Delta p_{t+1}) = 0 \text{ and } p_{t-1} = \frac{\pi\delta}{r}$$

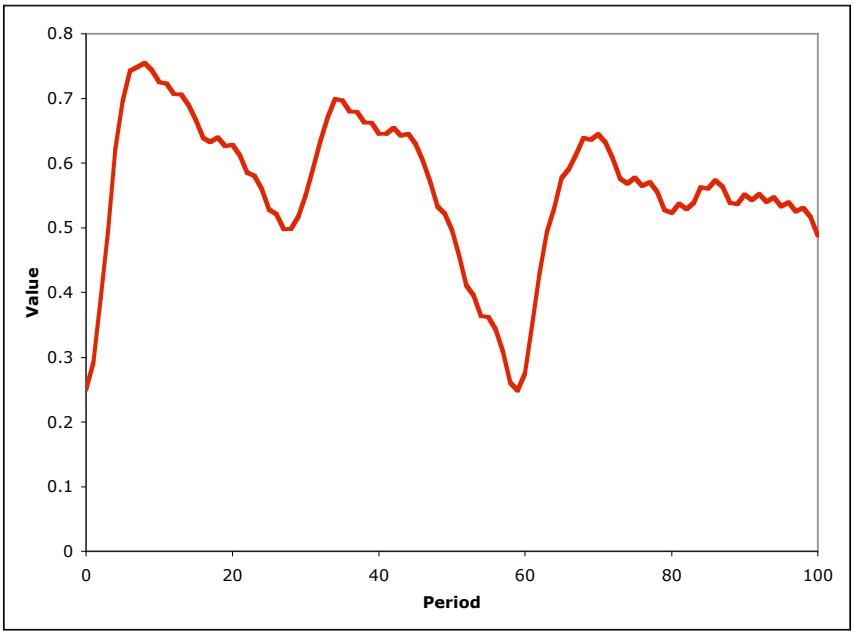
is the "fundamental" value of p: call it p*.

How does this model behave? Set some parameter values: set the probability π that the dividend is paid to 0.5, the value of the dividend when paid δ at .05, and the value of the interest rate at .05—so that the fundamental value p* is 0.5. Set the value of λ at 1.5: for each agent who makes a comparison, a number of agents equal to 1.5 times the last-period difference in rates of return actually switch their portfolio strategies. And set the initial price at 0.25—half the fundamental—to model Minsky's and

Kindleberger's idea of the "displacement": the start of the cycle by an initial gap between the past and the fundamental value of assets.

Then a typical simulation run of this simplest behavioral bubble model is given by Figure 2.

Figure 2
Stock Prices Over Time, λ=1.5

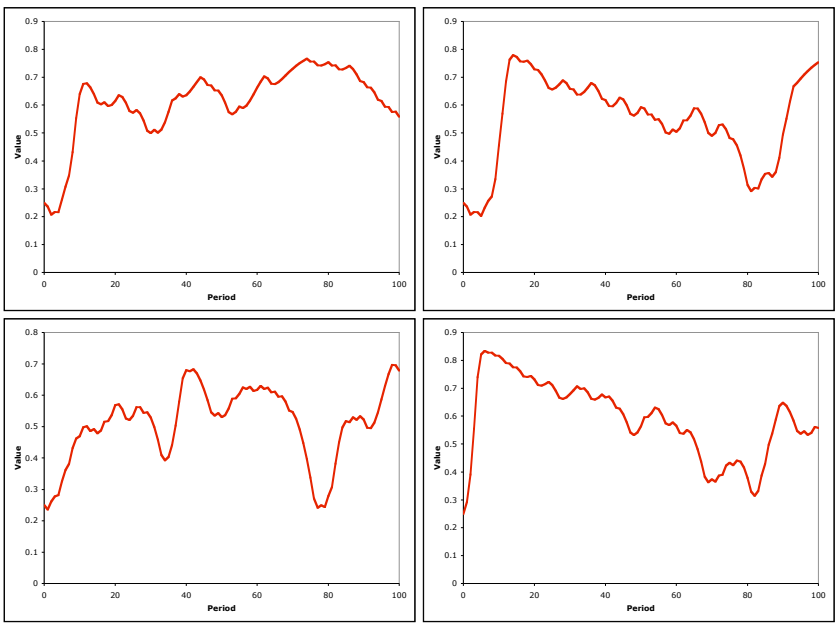


There are definitely manias—the large initial runup in prices as they roar past their fundamental value of 0.5 and keep going: the conversion of additional agents to stocks boosts demand for stocks, boosts stock prices, creates high returns, and so induces yet more

agents to convert to stocks which drives prices even higher until the market tops out at a price of 0.75, 50% above fundamentals, when a combination of few agents left who could convert to stocks plus a couple of zero dividend realizations sets the price on a downward trajectory. The high price means that dividend yields are low, and cannot compete with bond rates.

This run is not at all unrepresentative, as Figure 3 shows.

Figure 3
Four Additional Stock Price Simulation Runs, λ=1.5



Note that although we have manias we do not have panics and crashes. We have overshooting on both the upside and downside,

for the conversion rule is symmetrical between stocks and bonds. But there is no sense that the rush for the exit is more rapid than the rush for the entrance was. Indeed, the rush for the exit in this model is typically rather slow. A stock investor has to meet a bond investor before he or she can think about switching to bonds. The thing that stops the price runup is when there are few bond investors to switch to stocks and drive a further price rise. And that is also a situation in which there is little opportunity for stock investors to learn that they should switch to bonds.

This is a defect in the model: this tells us that we would need more than random meeting and imitation of successful strategies to get the "panics and crashes" part of manias, panics, and crashes.

Some insight can be gained if we rewrite (2) as:

$$(4) \qquad \left(\frac{p_{t+1}}{p_t} - 1\right) = \lambda(1 - p_t)\left(\frac{p_t}{p_{t-1}} - 1\right) + \lambda(1 - p_t)\left(\frac{d_t}{p_{t-1}} - r\right)$$

and compare it to what we get if we suppress the "speculative' term from (2)—allow for imitation depending not on price changes and capital gains but just on the comparison of the dividend yield to the bond interest rate:

$$(5) \qquad \left(\frac{p_{t+1}}{p_t} - 1\right) = \lambda(1 - p_t)\left[\left(\frac{d_t}{p_{t-1}}\right) - r\right]$$

Equation (5) is well-behaved: if the dividend yield was greater than r last period, investors flood in and the growth rate of the price is positive; if the dividend yield was less than r, investors exist and

the growth rate of the price is negative. Equation (4), however, has the extra positive-feedback term $\lambda(1-p_t)(p_t/p_{t-1} - 1)$ that imparts momentum to price changes. Indeed, whenever $(1-p_t)$ is near one the excess of the dividend over the bond yield drives not the rate of change of prices—not their velocity—but rather the rate of change of the rate of change of prices—their acceleration—and we have a system that bears a family relationship to a harmonic oscillator.

In an earlier era, we would have termed those who buy and sell stocks according to (5) "investors" and those who buy and sell according to (4) "speculators." And we would have written things like:

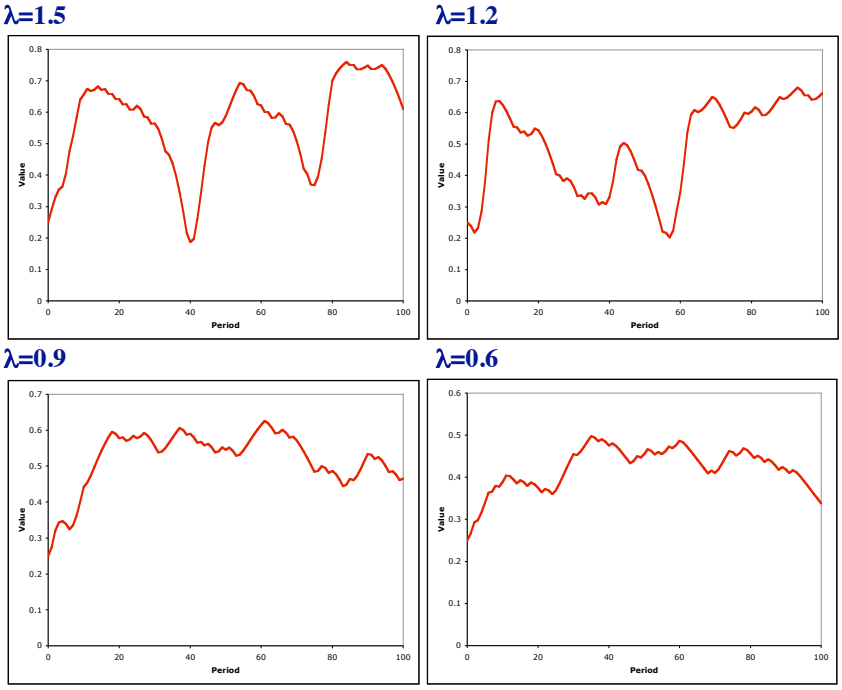
> If I may be allowed to appropriate the term speculation for the activity of forecasting the psychology of the market, and the term enterprise for the activity of forecasting the prospective yield of assets over their whole life, it is by no means always the case that speculation predominates over enterprise. As the organisation of investment markets improves, the risk of the predominance of speculation does, however, increase. In one of the greatest investment markets in the world, namely, New York, the influence of speculation (in the above sense) is enormous. Even outside the field of finance, Americans are apt to be unduly interested in discovering what average opinion believes average opinion to be; and this national weakness finds its nemesis in the stock market. It is rare, one is told, for an American to invest, as many Englishmen still do, "for income"; and he will not readily purchase an investment except in the hope of capital appreciation. This is only another way of saying that, when he purchases an investment, the American is attaching his hopes, not so much to its prospective yield, as to a favourable change in the conventional basis of valuation, i.e. that he is, in the above sense, a speculator. Speculators may do no harm as bubbles on

> a steady stream of enterprise. But the position is serious when enterprise becomes the bubble on a whirlpool of speculation. When the capital development of a country becomes a by-product of the activities of a casino, the job is likely to be ill-done. The measure of success attained by Wall Street, regarded as an institution of which the proper social purpose is to direct new investment into the most profitable channels in terms of future yield, cannot be claimed as one of the outstanding triumphs of laissez-faire capitalism — which is not surprising, if I am right in thinking that the best brains of Wall Street have been in fact directed towards a different object.

A market in which imitation and conversion are governed by (5) would seem reasonably likely to perform reasonably well as a social capital allocation and forecasting mechanism. A market in which imitation and conversion are governed by (4) would not.

Thus if we have short look-back periods for comparing returns, poor general knowledge of long-run fundamentals, a tendency to adopt recently-successful strategies, and moderately long-duration assets traded at some frequency then it is easy to generate bubbles and manias and overshooting on the upside and on the downside—if the imitation parameter λ is large enough. A smaller imitation parameter λ will first reduce and then eliminate market misbehavior via mania in the same way that turning down the volume dial will eliminate feedback in the auditorium. This is seen in Figure43, which shows sample simulation runs as the parameter λ is tuned from 1.5 down to 0.6. Of course, with a lower imitation parameter it takes the market much longer to adjust the price to its new equilibrium if there is a change in underlying fundamentals. But at least the possibility of positive-feedback market meltdown is eliminated.

Figure 4
Shrinking the Imitation Parameter λ



Although we now have manias, bubbles, and overshootings, this is not what we really want. Panics and crashes are asymmetrically-strong *downward* movements. We cannot generate them out of a model that is, after all, a symmetric one.

In order to get financial panics and crashes we are going to have to complicate the model and make stock holdings "fragile" at the peak. Without eliminating the positive-feedback mechanisms that produce the runup to the bubble peak and then the cessation of the

upward movement as the supply of bigger fools to buy stocks is exhausted, we have to make it so that a small price decline will trigger a massive wave of selling. In short, we have to introduce the margin call and the forced sale into the model. There has to be asymmetry: profits make you feel exuberant—rationally or irrationally—and prone to expand your position; but losses leave you bankrupt and your position is then involuntarily sold out from under you. We have to introduce portfolio insurance, stop-loss orders, margin calls, capital requirements, and many other contractual mechanisms work to make positive-feedback trading on the downside automatic and hence swift, while positive-feedback trading on the upside remains discretionary.

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